

## Statistical Mechanics 219 SQ 2018

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Bonus Homework  
Solve ASAP

1. The free energy of a system as a function of temperature and volume is given by  $F = -AT^2V^3$ .
  - (a) Compute the entropy  $S$  as a function of temperature  $T$  and volume  $V$ .
  - (b) Compute pressure  $P$  as a function of temperature and volume.
  - (c) The pressure is increased from  $P_1$  to  $P_2$  isothermally at temperature  $T$ . Compute the amount of heat received by the system. (The amount of heat should be a function of  $P_1$ ,  $P_2$  and  $T$  only).
2. The thermodynamic potential of a system is given by  $\Phi = -AT^2P + BP$ , where  $T$  and  $P$  are pressure and  $A$  and  $B$  are constants.
  - (a) Compute the entropy  $S$  as a function of temperature  $T$  and pressure  $P$ .
  - (b) Compute volume  $V$  as a function of pressure  $P$  and temperature  $T$ .
  - (c) The system is expanding isothermally at temperature  $T$  from volume  $V_1$  to volume  $V_2$ . Compute the amount of heat it absorbs.
3. In a big reservoir filled with a classical ideal gas there is a hole of the area  $S$ , through which the gas leaks slowly into a vacuum. The hole is very small (smaller than the mean free path of the molecules inside), so that the velocity distribution of the gas inside is unaffected by the leakage. The concentration of the molecules is  $n$ , the velocity distribution function is  $f(v)$ . Compute the flux of the particles through the hole.
4. In a big reservoir filled with a classical ideal gas there is a tiny hole of the area  $S$ , through which the gas leaks slowly into a vacuum. Compute the jet force with which the leaking gas propels the reservoir. The concentration of the molecules, their mass and temperature are known.
5. Electrons in the surface layer of a semiconductor may often be considered as an ideal 2D gas. Compute how many times a unit length of the perimeter of this gas gets hit by electrons per time, if the effective mass of an electron is  $m$ , the temperature is  $T$  and the concentration is  $n$ .
6. A gas of molecules with mass  $m$  is confined in a cylinder of height  $L$  and exposed to the potential  $U(z) = az^2$  which varies only along the  $z$  axis, i.e. the height of the cylinder. The temperature of the gas is  $T$ . Compute the position of the centre of mass of the gas.
7. Compute the pressure of a five-dimensional (5D) ideal gas at temperature  $T$  by calculating the momentum that the molecules of this gas transfer to a wall of unit area per time. The molecules have a quadratic dispersion with the mass  $m$ . The concentration is  $n$ , the temperature is  $T$ .

8. The concentration of  $^{40}\text{Ar}$  near the surface of the Earth is 0.9%. At some altitude, the pressure of the atmosphere is 10 times smaller than at the surface. What would be the concentration of  $^{40}\text{Ar}$  at this altitude if the atmosphere was isothermal?
9. Who has more air: a 10cm-thick layer of air near the surface of the Earth or a 1km-thick layer at the altitude  $L = 100\text{km}$ ? Assume that the atmosphere is isothermal at  $T = 300\text{K}$ .